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SHORTER COMMUNICATIONS

A CLOSED-FORM SOLUTION TO RADIANT INTERCHANGE BETWEEN NONISOTHERMAL PLATES

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INTRODUCTION

RADIANT interchange between diffuse surfaces is described by Fredholm integral equations. Usually [1–3] these integral equations are solved by numerical methods or approximate analytical techniques. Closed form solutions [1] have been obtained only for the spherical cavity and the circular-arc cavity. The primary objective of this note is to present a closed-form solution for the parallel plate geometry. The secondary objective is to study the effect of temperature fluctuations on radiant interchange.

PROBLEM AND SOLUTION

In an attempt to investigate the effect of temperature fluctuations on the radiative interchange between surfaces, consider two infinite plates separated by a distance h [see Fig. 1(a)]. The assumed nonisothermal temperature distribution on the plates is illustrated on Fig. 1(b) and is defined by equation (1)

$$T(y) = T_r + T_0 \cos \beta y \tag{1}$$

where T_r is some reference temperature, T_0 is proportional to the maximum temperature variation, $T_r > T_0$ and β is a parameter. While temperature distribution (1) is relatively simple, it is representative of a fluctuating temperature distribution. Notice the plates are opaque, diffuse, and have identical properties.

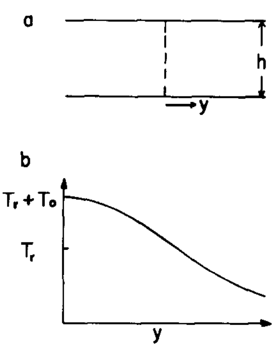

FIG. 1. (a) Schematic of geometry of consideration. (b) Nonisothermal temperature profile.

The general expression for the radiosity is presented in the book by Love [2] and for the infinite plate geometry it becomes

$$R(y) = \varepsilon\sigma T^4(y) + \frac{\rho}{2}\int_{-\infty}^{\infty} \frac{R(\bar{y})h^2\,\mathrm{d}\bar{y}}{[(y-\bar{y})^2+h^2]^{3/2}}. \tag{2}$$

when the symmetry of the problem is taken into account. In this form five parameters (ρ, β, h, T_0, T_r) must be specified before the radiosity equation can be solved. Using standard trigonometric identities, $T^4(y)$ may be cast in the following form:

$$T^4(y) = T_r^4 + 3T_r^2T_0^2 + \tfrac{3}{8}T_0^4 + (4T_r^3T_0 + 3T_rT_0^3)\cos\beta y + (3T_r^2T_0^2 + \tfrac{1}{2}T_0^4)\cos 2\beta y + T_rT_0^3\cos 3\beta y + \tfrac{1}{8}T_0^4\cos 4\beta y. \tag{3}$$

Since each nonhomogeneous term is in the cosine form and equation (2) is linear, the radiosity can be expressed as

$$R(y) = \varepsilon\sigma(T_r^4 + 3T_r^2T_0^2 + \tfrac{3}{8}T_0^4)B(y,0) + \varepsilon\sigma(4T_r^3T_0 + 3T_rT_0^3)B(y,\beta) + \varepsilon\sigma(3T_r^2T_0^2 + \tfrac{1}{2}T_0^4) \times B(y,2\beta) + \varepsilon\sigma T_rT_0^3B(y,3\beta) + \frac{\varepsilon\sigma}{8}T_0^4B(y,4\beta) \tag{4}$$

where the non-dimensional radiosity $B(y,\beta)$ satisfies the following integral equation:

$$B(y,\beta) = \cos\beta y + \frac{\rho}{2}\int_{-\infty}^{\infty}\frac{B(\bar{y},\beta)h^2}{[(y-\bar{y})^2+h^2]^{3/2}}\,\mathrm{d}\bar{y}. \tag{5}$$

Note the function $B(y,\beta)$ depends on only three parameters (ρ, β, h). The solution to equation (5) may be determined directly by assuming the following form for $B(y,\beta)$:

$$B(y,\beta) = A\cos\beta y. \tag{6}$$

After substitution of equation (6) into equation (5) along with the variable change of $y^* = \bar{y} - y$, and execution of the simple integration [4] the solution is

$$B(y,\beta) = \frac{\cos\beta y}{1-\rho h\beta K_1(h\beta)} \tag{7}$$

where K_1 is the modified Bessel function of the second kind of order one.

The expression for the local nondimensional heat transfer (radiative flux) is presented by Love [2] and is given

$$\bar{q}(y,\beta) = \frac{1}{\rho}[\cos\beta y - \varepsilon B(y,\beta)] = \cos\beta y\,\frac{1-h\beta K_1(h\beta)}{1-\rho h\beta K_1(h\beta)}. \quad (8)$$

Now the heat transfer for the aforementioned problem can be expressed in terms of $\bar{q}(y,\beta)$, i.e.

$$q(y) = \varepsilon\sigma\{(4T_r^3T_0+3T_rT_0^3)\bar{q}(y,\beta)+(3T_r^2T_0^2+\tfrac{1}{2}T_0^4)\bar{q}(y,2\beta) + (T_rT_0^3)\bar{q}(y,3\beta)+(T_0^4/8)\bar{q}(y,4\beta)\}. \quad (9)$$

The overall nondimensional heat transfer is determined by

$$\bar{Q}(\beta) = \int_{-\frac{\pi}{\beta}}^{\frac{\pi}{\beta}} \bar{q}(y,\beta)\,dy \quad (10)$$

since,

$$B(y,\beta) = B\left(y+\frac{2\pi}{\beta},\ \beta\right),$$

only the average over one cycle need be determined. The computation indicated by equation (10) results in an answer of zero. Thus the overall heat transfer for the aforementioned problem is

$$Q = \varepsilon\sigma\left\{(4T_r^3T_0+3T_rT_0^3)\bar{Q}(\beta)+(3T_r^2T_0^2+\tfrac{1}{2}T_0^4)\bar{Q}(2\beta) + (T_rT_0^3)\bar{Q}(3\beta)+\left(\frac{T_0^4}{8}\right)\bar{Q}(4\beta)\right\} \equiv 0. \quad (11)$$

RESULTS AND CONCLUSIONS

The behavior of the nondimensional radiosity, $\varepsilon B(y,\beta)$, at $y = 0$ is presented in Fig. 2. The radiosity varies from unity for the isothermal case ($\beta = 0$) to ε for large $h\beta$. A surface with a nondimensional radiosity of unity appears black, while a nondimensional radiosity of ε implies a gray surface with zero irradiation. Thus, for $h\beta > 5$ the surface at $y = 0$ receives no energy from the other surface. Physically viewing the rapid spatial fluctuations ($h\beta > 5$) on the other surface from $y = 0$, one sees only an average of these fluctuations which is zero. For $\rho < 0{\cdot}1$, the radiosity depends weakly on $h\beta$ and approximates that of a black surface. Near $\rho = 1$ the nondimensional radiosity is strongly dependent on ε.

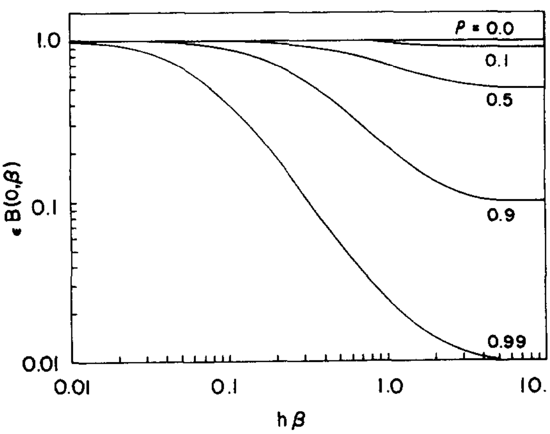

FIG. 2. Nondimensional radiosity.

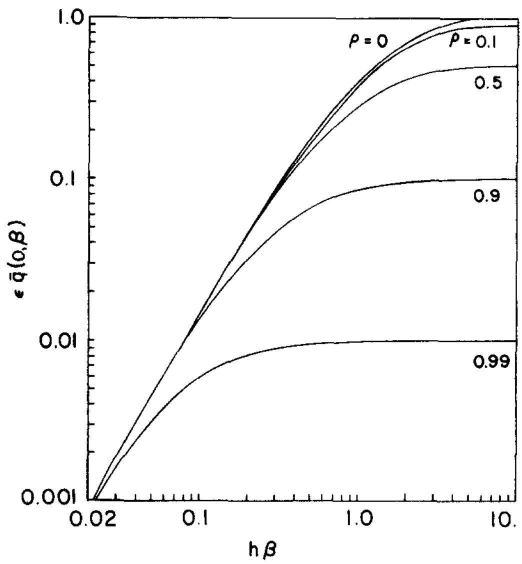

FIG. 3. Nondimensional local heat transfer.

The nondimensional local heat transfer, $\varepsilon\bar{q}(y,\beta)$, at $y = 0$ is illustrated in Fig. 3. The local heat transfer varies from zero at $\beta = 0$ to ε for large $h\beta$. As with the radiosity, the local heat transfer is independent of $h\beta$ for $h\beta > 5$. For $h\beta \ll 1$, the local heat transfer is independent of ρ.

From an experimental point of view the interesting result of this investigation is the fact that for nonisothermal surfaces, the overall heat transfer may be zero, while the local heat transfer may execute large variations with position.

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